# AVAILABILITY ANALYSIS OF A TWO-UNIT STANDBY SYSTEM WITH DELAYED REPLACEMENT UNDER PERFECT SWITCHING

By

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ABSTRACT: In this paper, the author has initiated the study of availability of the system which consists of two identical cold standby units with constant failure rates. Initially, one unit is operative while other remains as standby. Each of the units of the system has three modes, i.e., normal, degraded and total failure. The system fails when both the units fail totally and may also fail due to common cause failure. The time taken in replacement of a failed unit by a standby unit is not negligible but is a random variable.
KEYWORDS: Standby system, Repairable system, Supplimentry variable.

INTRODUCTION : The author has considered here, a complex system having two identical cold standby units with constant failure rates. Initially, one unit is operative while other remains as standby. Each of the units of the system has three modes. i.e., normal, degraded and total failure. The system fails when both the units fail totally and may also fail due to common cause failure. The time taken in replacement of a failed unit by a standby unit is not negligible but is a random variable. Failure time distribution of the units are exponential while inspection rates, replacement rate and repair rate time distributions are quite general. Reliability parameters, generally mean time to systems failure (MTSF) and availability in such systems have also been obtained using the theory of Semi-Markov process and supplementary variable process. The purpose of the present chapter is to discuss a two-unit cold standby with three modes dissolving the above two very stringent assumptions.

**SYSTEM DESCRIPTION** :

(i) A cold standby system comprises two similar units. Each unit has three modes: normal (N), Degraded (D) and total failure (F). The units are said to be N,D and F units if they are in their respective modes.

(ii) The replacement time of a failed unit is not negligible but is a random variable called delay time.

(iii) The system fails on the total failure of its both units and also breaks down completely due to common cause failure.

(iv) In the normal state, the system is inspected with the general inspection rate distribution.

(v) Failure time distributions are negative exponential, while the repair and delay time distributions are arbitrary.

(vi) A single service facility is available to repair a D-unit, a F-unit and to activate the cold standby unit. The repair facility is not always with the system but can be made available instantaneously whenever needed.

(vii) Initially system works with normal efficiency after inspection.

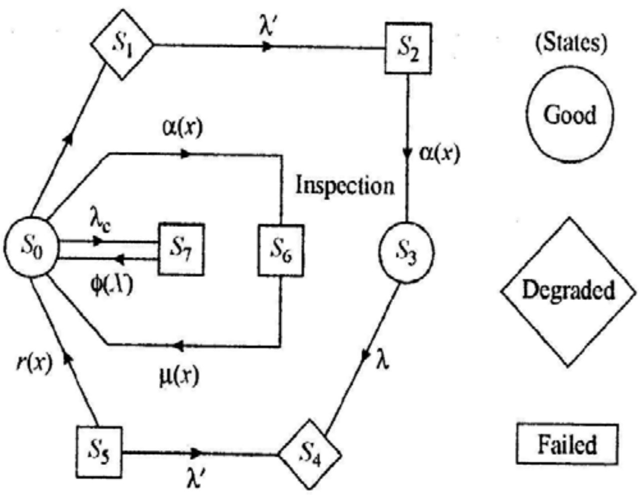
(viii) A repaired unit works as good as new.

**NOTATIONS:**

| | |
|---|---|
| $D / D_t / D_x$ | : $\frac{d}{dt} / \frac{\partial}{\partial t} / \frac{\partial}{\partial x}$ |
| $\lambda / \lambda'$ | : Constant failure rates from N to D/D to F |
| $\lambda_c$ | :Constant common cause failure rate |
| $\alpha(x) / \mu(x)$ | : Inspection rate from $S_0$ to $S_6 / S_6$ to $S_0$ |
| $r(x) / \phi(x)$ | : General repair rate from $S_5$ to $S_0 / S_7$ to $S_0$ |
| $\beta(x)$ | : Switching rate from $S_2$ to $S_1$ |
| $P_i(t)$ | : Probabilities in the $S_i$ state where i= 1, 3, 4 |

$P_2(x,t)$ : Probabilities at the $S_i$ state where i = 0, 2, 5, 6, 7.

**TRANSITION DIAGRAM**:




**FORMULATION OF MATHEMATICAL PROBLEM** : Viewing the nature of this system, we obtain the following set of difference differential equations by connecting the state probabilities at time $t+\Delta$ with those at time $t$ and taking limits as $\Delta t \to 0$.

$$\left[D_x + D_t + \lambda + \lambda_c + \alpha(x)\right] P_0(x,t) = 0 \tag{1}$$

$$\left[D + \lambda'\right] P_1(t) = \lambda P_0(t) \tag{2}$$

$$\left[D_x + D_t + \beta(x)\right] P_2(x,t) = 0 \tag{3}$$

$$\left[D + \lambda\right] P_3(t) = \int_0^\infty \beta P_2(x,t)\,dx \tag{4}$$

$$\left[D + \lambda\right] P_4(s) = \lambda P_3(t) \tag{5}$$

$$\left[D_x + D_t + r(x)\right] P_5(x,t) = 0 \tag{6}$$

$$\left[D_x + D_t + \mu(r)\right] P_6(x,t) = 0 \tag{7}$$

$$\left[D_x + D_t + \phi(x)\right] P_7(x,t) = 0 \tag{8}$$

These equations are to be solved under following boundary and initial conditions:

**Boundary Conditions :**

$$P_0(0,t) = \int_0^\infty \bar{P}_5(x,t) r(t)\,dx + \int_0^\infty P_7(x,t)\phi(x)\,dx + \int_0^\infty P_6(x,t)\mu(x)\,dx \tag{9}$$

$$P_2(0,t) = \lambda' P_1(t) \tag{10}$$

$$\bar{P}_5(0,t) = \lambda' P_4(t) \tag{11}$$

$$\bar{P}_6(0,t) = \int_0^\infty P_0(x,t)\alpha(x)\,dx \tag{12}$$

$$\bar{P}_7(0,t) = \lambda_c P_0(t) \tag{13}$$

**Initial Conditions :**

$$P_0(0) = 1$$

$$P_k(0) = 0 \quad \forall (k \neq 0)$$

**SOLUTION OF THE PROBLEM :** Taking Laplace transform of equations (1) through (13) and solving one by one, using initial conditions and after minor simplification, one may obtain

$$\bar{P}_0(s) = \frac{B(s)}{A(s)} \tag{14}$$

$$\bar{P}_1(s) = \frac{\lambda}{s+\lambda'} \cdot \frac{B(s)}{A(s)} \tag{15}$$

ISSN 2229-5518

$$\overline{P}_2(s)=\frac{\lambda\lambda'}{s+\lambda'}.\frac{B(s)}{A(s)}.D_\beta(s) \tag{16}$$

$$\overline{P}_3(s)=\frac{\lambda\lambda'}{(s+\lambda)(s+\lambda')}.\frac{B(s)}{A(s)}.\overline{S}^\beta(s) \tag{17}$$

$$\overline{P}_4(s)=\frac{\lambda^2\lambda'}{(s+\lambda)(s+\lambda)^2}.\frac{B(s)}{A(s)}.\overline{S}^\beta(s) \tag{18}$$

$$\overline{P}_5(s)=\frac{\lambda^2\lambda'^2}{(s+\lambda')^2(s+\lambda)}.\frac{B(s)}{A(s)}.\overline{S}^\beta(s).D_r(s) \tag{19}$$

$$\overline{P}_6(s)=\frac{\overline{S}^\alpha(s+\lambda+\lambda_c)}{A(s)}.D_\mu(s) \tag{20}$$

$$\overline{P}_7(s)=\frac{\lambda_c}{\lambda}.\frac{B(s)}{A(s)}.D_\phi(s) \tag{21}$$

Where

$$A(s)=1\frac{\lambda^2\lambda'^2}{(s+\lambda)(s+\lambda')^2}.\overline{S}^\beta(s)-\overline{S}^r(s)-\overline{S}^\alpha(s+\lambda+\lambda_c).\overline{S}^\mu(s)-\lambda_c B(s)\overline{S}^\phi(s)$$

$$B(s)=D_\alpha(s+\lambda+\lambda_c)$$

$$D_\theta(s)=\frac{1-\overline{S}^\theta(s)}{s}$$

**EVALUATION OF LAPLACE TRANSFORMS OF PROBABILITIES UP AND DOWN STATES :**

$$\overline{P}_{UP}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_3(s)+\overline{P}_4(s)$$

$$=\frac{B(s)}{A(s)}\left[1-\frac{\lambda}{s+\lambda'}+\frac{\lambda\lambda'}{(s+\lambda)(s+\lambda')}\overline{S}^\beta(s)+\frac{\lambda^2\lambda'}{(s+\lambda)(s+\lambda')}.\overline{S}^\beta(s)\right] \tag{22}$$

$$\overline{P}_{down}(s)=\overline{P}_2(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_7(s)=\frac{1}{s}-P_{up}(s) \tag{23}$$

It is worth noticing that

$$\overline{P}_{up}(s)+\overline{P}_{down}(s)=\frac{1}{s} \tag{24}$$

**ERODIC BEHAVIOUR OF THE SYSTEM :** Using Abel's Lemma $viz., \lim_{s\to 0} S\overline{F}(S)=\lim_{t\to\infty} F(t)=F(say)$ ; provided limit on R.H.S. exists, the following time independent probabilities have been obtained from equations (14) through (22).

$$P_0=\frac{B(0)}{A'(0)} \tag{25}$$

$$P_1=\frac{\lambda}{\lambda'}.\frac{B(0)}{A'(0)} \tag{26}$$

$$P_2=\lambda.\frac{B(0)}{\lambda'(0)}M^\beta \tag{27}$$

$$P_3=\frac{B(0)}{A'(0)} \tag{28}$$

$$P_4=\frac{\lambda}{\lambda'}.\frac{B(0)}{A'(0)} \tag{29}$$

$$P_5=\lambda.\frac{B(0)}{A'(0)}M^r \tag{30}$$

$$P_6=\frac{\overline{S}^\alpha(\lambda+\lambda_c)}{A'(0)}M^\mu \tag{31}$$

$$P_7 = \frac{\lambda_c}{\lambda} \cdot \frac{B(0)}{A'(0)} M^{\phi} \quad (32)$$

$$P_{up} = A_{vss} = \frac{2B(0)}{A'(0)}\left[1+\frac{\lambda_t}{\lambda}\right] \quad (33)$$

Where $B(0) = \left[\beta(s)\right]_{s=0}$

$$A'(0) = \left[\frac{d}{ds} A(s)\right]_{s=0}$$

$A_{vss}$ = Availability at $t = \infty$

And $M'(i = \beta, r, \mu, \phi) = \bar{S}^{\alpha}(0)$

**PARTICULAR CASES :** (When repair follows exponential time distribution)

setting $\bar{S}^{x}(s) = \frac{x}{s+x}$

Where $x = \beta, r, \phi, \alpha, \mu$ in equations (14) through (21), we get

$$\bar{P}_0(s) = \frac{D(s)}{C(s)} \quad (34)$$

$$\bar{P}_1(s) = \frac{\lambda}{(s+\lambda)} \cdot \frac{D(s)}{C(s)} \quad (35)$$

$$\bar{P}_2(s) = \frac{\lambda\lambda'}{(s+\lambda)(s+\beta)} \cdot \frac{D(s)}{C(s)} \quad (36)$$

$$\bar{P}_3(s) = \frac{\lambda\lambda'}{(s+\lambda)(s+\lambda')} \cdot \frac{D(s)}{C(s)} \cdot \frac{\beta}{(s+\beta)} \quad (37)$$

$$\bar{P}_4(s) = \frac{\lambda^2\lambda'}{(s+\lambda)(s+\lambda')^2} \cdot \frac{D(s)}{C(s)} \cdot \frac{\beta}{(s+\beta)} \quad (38)$$

$$\bar{P}_5(s) = \frac{\lambda^2\lambda'^2}{(s+\lambda)(s+\lambda')^2} \cdot \frac{D(s)}{C(s)} \cdot \frac{\beta}{(s+\beta)} \cdot \frac{2}{s+r} \quad (39)$$

$$\bar{P}_6(s) = \frac{\alpha}{(s+\alpha+\lambda+\lambda_c)} \cdot \frac{1}{C(s)} \cdot \frac{1}{s+\mu} \quad (40)$$

$$\bar{P}_7(s) = \frac{\lambda_c}{\lambda} \cdot \frac{D(s)}{C(s)} \cdot \frac{1}{s+\phi} \quad (41)$$

Where

$$C(s) = 1 - \frac{\lambda^2\lambda'^2 D(s)}{(s+\lambda)(s+\lambda')^2} \cdot \frac{\beta}{(s+\beta)} \cdot \frac{r}{(s+r)} - \frac{\alpha\mu}{(s+\alpha+\lambda+\lambda_c)(s+\mu)} - \lambda_c D(s) \frac{\phi}{(s+\phi)}$$

$$D(s) = \frac{1}{s+\alpha+\lambda+\lambda_c}$$

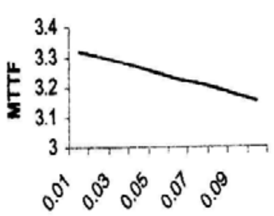
**RELIABILITY AND MTTF :**

$$\bar{R}(s) = \frac{1}{(s+\alpha+\lambda+\lambda_c)}\left[1+\frac{\lambda}{s+\lambda'}\right] \quad (42)$$

$$MTTF = \lim_{s\to 0} \bar{R}(s) = \frac{1}{(\alpha+\lambda+\lambda_c)}\left[1+\frac{\lambda}{\lambda'}\right] \quad (43)$$

For a complex configuration having parametric values

$\lambda = 0.01$, $\lambda' = 0.02$, $\alpha = 0.8$



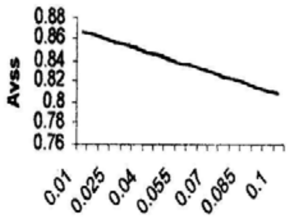




Fig.1. MTTF vs Failure Fig. 2. Avss vs Failure

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